Reinforcement Learning from Human Feedback (Preference-based RL)

Classical RL with a reward function

In standard RL, we assume a scalar reward function $r(s, a)$ for each state and action, with cumulative return:

$$R(\tau) = \sum_{t=1}^{H} r(s_h, a_h).$$

- The trajectory τ is a sequence of H state-action pairs $(s_1, a_1, s_2, a_2, \cdots, s_H, a_H)$.
- This works well when the reward function truly captures the task objective.
- In many real problems, we only have a hand-crafted reward **proxy** for what we actually want (e.g., goal + reward shaping).

Reward misspecification and reward hacking

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The handcrafted reward fails to capture the true objective.

Reward hacking

The agent identifies a strategy that maximizes the proxy reward but fails to accomplish the task.

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RL often fails because optimization is **too successful on the wrong reward**.

Examples of reward hacking in classical RL

Example

- **Navigation/robotics:** We reward progress to the goal, but the agent learns to trigger the progress bonuses repeatedly without reaching the goal.

Challenge

It is hard to design a scalar reward to capture safety, robustness, and human satisfaction.

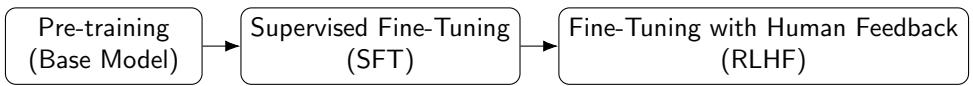
Preference-based RL

Agents should learn from human feedback rather than from handcrafted reward signals.

A "Famous" Example

GPT 3

Both the base model and SFT are based on supervised learning, and RLHF is based on reinforcement learning.



What is a GPT Transformer?

Next token prediction: Example

The cat sat "?"

GPT Transformer

- Step 1: Input text $\Rightarrow$ tokens $\Rightarrow$ token embeddings and position embeddings
- Step 2: Embedding $\Rightarrow$ Transformer block 1 $\Rightarrow$ Transformer block 2 $\cdots$ Transformer block L.
- Step 3: Linear layer $\Rightarrow$ Softmax over vocabulary $\Rightarrow$ Next-token distribution

GPT Transformer: Step 1

Tokenization and Embedding: Example

Word	Token	Embedding
The	101	$[0.2,\ -1.1,\ 0.7,\ 0.4]^\top$
Cat	572	$[-0.5,\ 0.3,\ 1.4,\ -0.8]^\top$
Sat	913	$[1.0,\ 0.1,\ -0.6,\ 0.9]^\top$

GPT Transformer: Step 1

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Why embedding?

We can measure similarity and closeness of two words based on the embedding vectors.

GPT Transformer: Step 1

Position embedding

Position	Position Embedding
1	$[0.01,\ 0.02,\ 0.03,\ 0.04]^\top$
2	$[0.05,\ 0.06,\ 0.07,\ 0.08]^\top$
3	$0.09,\ 0.10,\ 0.11,\ 0.12]^\top$

Why position embedding?

Tell the model where a token is. So “cat sat” and “sat cat” are different.

GPT Transformer: Step 1

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Word (x_i)	Embedding h_i
The	$[0.21,\ -1.08,\ 0.73,\ 0.44]^\top$
Cat	$[-0.45,\ 0.36,\ 1.47,\ -0.72]^\top$
Sat	$[1.09,\ 0.2,\ -0.49,\ 1.02]^\top$

GPT Transformer: Step 2

Self Attention (Query, Key, Value)

- Query: information I am looking for.

$$Q = HW_Q = \begin{pmatrix} 0.21 & -1.08 & 0.73 & 0.44 \\ -0.45 & 0.36 & 1.47 & -0.72 \\ 1.09 & 0.2 & -0.49 & 1.02 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1.73 & -0.14 \\ -1.53 & 1.38 \\ 1.91 & 0.80 \end{pmatrix}$$

- Key: information I have

$$K = HW_K = \begin{pmatrix} 0.21 & -1.08 & 0.73 & 0.44 \\ -0.45 & 0.36 & 1.47 & -0.72 \\ 1.09 & 0.2 & -0.49 & 1.02 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 1 \\ 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 3.54 & -0.79 \\ -0.42 & 2.55 \\ 1.22 & -1.31 \end{pmatrix}$$

GPT Transformer: Step 2

Score Matrix

$$S = \frac{QK^\top}{\sqrt{d_k}} = \frac{1}{\sqrt{d_k}} H W_Q W_K^\top H^\top$$

where $S(i, j)$ is the relevance of token j to token i.

Attention-Weight Matrix and Causal Masking

$A = [A_{ij}]$ such that

$$A_{ij} = \frac{e^{S_{ij}}}{\sum_{m \le i} e^{S_{im}}} \quad j \le i; \quad A_{ij} = 0 \quad j > i.$$

Token i can only attend to earlier tokens.

GPT Transformer: Step 2

Attention-Weight Matrix and Causal Masking: Example

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0.3 & 0.7 & 0 \\ 0.2 & 0.5 & 0.3 \end{pmatrix}$$

Output of Attention

Value $V = HW_V$ is the information to be retrieved. The output of attention is

$$Z = AVW_O,$$

where W_O map the output a $n \times d$ matrix (the same as H).

Let $H \leftarrow H + AVW_O$ and pass it to the next attention block.

GPT Transformer: Step 3

Linear layer and next-token distribution

Input the representation of the last token $h_n^{(L)}$ to a a feed-forward network. The output (logit) is

$$o_n = h_n^{(L)} W_{FF} + b,$$

where the dimension of o_n is the vocabulary size. For example,

$$o_n = [1.2,\ 0.4,\ 3.8,\ -0.7]^\top.$$

Next token distribution

$$p_n = \text{softmax}(o_n) = \left[\frac{e^{o_{n,i}}}{\sum_j e^{o_{n,j}}} \right]^\top$$

GPT Transformer: Multi-head attention

Each attention block has multiple heads. Each head, say head m, has its own $W_Q^{(m)}$, $W_K^{(m)}$, and $W_V^{(m)}$. The outputs are concatenated and projected

$$\tilde{Z} = [A_1 V_1, \cdots, A_M V_M] \tilde{W}_O.$$

GPT Transformer: Training

Use the cross-entropy loss for next-token prediction. Given a training sequence $(x_1, x_2, \cdots, x_N)$. The loss is

$$\mathcal{L} = \frac{1}{N} \sum_t - \log P(x_{t+1} | x_1, \cdots, x_t)$$

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Base Model

Next token prediction with all Internet data.

SFT

Next token prediction with human demonstrations (prompt, ideal response). Each demonstration becomes a sequence $(x_1, x_2, \cdots, x_m)$.

Reference

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